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NRL Memorandum Report 3465

**A Brief Review of Some Recent Results that  
can Improve our Theoretical Understanding of  
Magnetic Field Reconnection and Thermalization as  
Applicable to Solar Flares**

**D. S. SPICER**

*E.O. Hulburt Center for Research*

March 1977



**NAVAL RESEARCH LABORATORY**  
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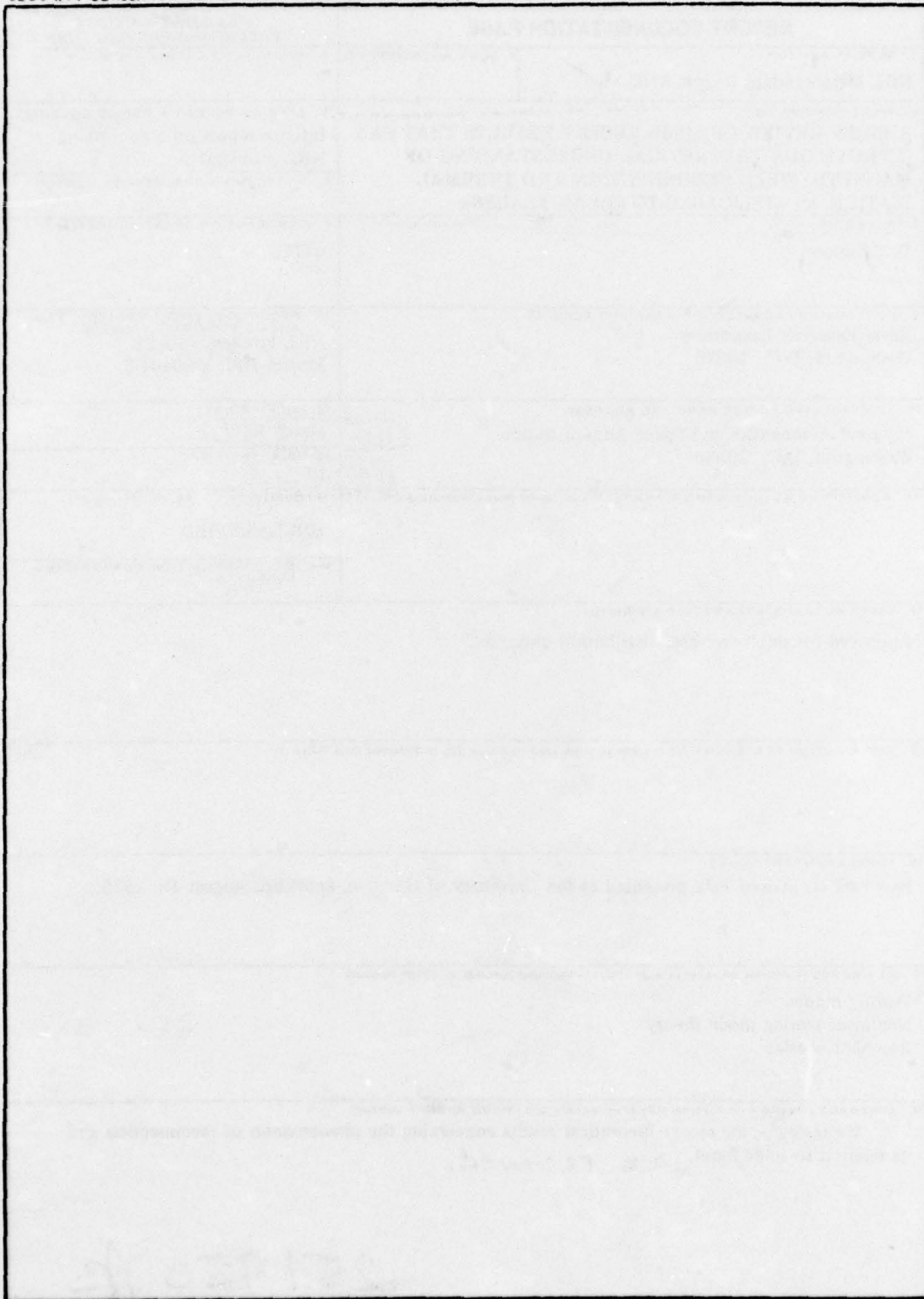
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**A BRIEF REVIEW OF SOME RECENT RESULTS THAT CAN IMPROVE  
OUR THEORETICAL UNDERSTANDING OF MAGNETIC FIELD RECONNECTION  
AND THERMALIZATION AS APPLICABLE TO SOLAR FLARES**

First, we should like to remind the reader of certain properties of the tearing mode; then we shall attempt to relate the tearing mode developed by Furth, Killeen, and Rosenbluth (FKR) (1963) with the work of Sweet (1958a,b), Parker (1963) and Petschek (1964).

The linear tearing mode, as originally developed by FKR, shows that the tearing mode can occur in any magnetic topology which possesses shear. The mode was found to be well localized around surfaces sometimes called singular surfaces, resonant surfaces\*, resistive layers, or mode rational surfaces where the quantity  $\underline{k} \cdot \underline{B}$  vanishes,  $\underline{k}$  being the wave vector of the perturbation and  $\underline{B}$  the equilibrium magnetic field. One should notice that a large number of surfaces can be excited if a spectrum of  $\underline{k}$  exists and  $|\underline{k}| a < 1$ , where  $a \approx (\nabla |\underline{B}| / |\underline{B}|)^{-1}$ . One should further note that because more than one surface can exist these surfaces can interact as will be discussed later. Also note that only one component of  $\underline{B}$  need vanish where  $\underline{k} \cdot \underline{B}$  vanishes, i.e., the total  $\underline{B}$  need not vanish. As an example, applicable to a current-carrying arch or filament (Spicer, 1976), the quantity  $\underline{k} \cdot \underline{B} = m B_\phi / r - 2 n B_T / R = B_\phi / r (m - n q(r))$ , changes sign at the mode rational surfaces  $q = m/n$ , where  $q = 2 r B_T / B_\phi R$ ,  $B_\phi$  and  $B_T$  are the poloidal and the toroidal components of  $\underline{B}$ , respectively, and  $R$  the global curvature of the arch. At these surfaces the tearing mode forms  $m$  x-type neutral points and  $m$  magnetic islands which result in current perturbations and current redistribution and the formation of current maxima and minima in the poloidal direction. (Figure 1). Further, the tearing mode is really a flute mode since where  $\underline{k} \cdot \underline{B}$  vanishes the components of the  $\underline{J} \times \underline{B}$  restoring force vanish, thereby permitting the continued growth of the

\*The use of the term "mode rational surface" is usually used when discussing closed field configurations, such as Tokamaks.

Note: Manuscript submitted February 17, 1977.

mode. The fact that the tearing mode is a flute mode is important when one looks for mechanisms that can excite the tearing mode (Spicer, 1976) and thereby trigger a flare.

As is well known, one of the first applications of the tearing mode to the flare phenomenon was by Sturrock (1966), in his now well-known helmet streamer model. There are, however, a number of theoretical difficulties with this model. Basically, the tearing mode by itself, cannot release enough magnetic energy; only the energy contained initially in the single sheet is converted, as pointed out by Priest (1975), since no new magnetic flux is inwardly convected. Note that Sturrock used only one sheet; however, he could have used a number of sheets by invoking a sheared field. Since no flux is convected in, Sturrock suggested that the tearing mode developed non-linearly into the Petschek mechanism. Is this a valid assumption? Kaw (1976) appears to have shown that it is. To see this we must look at recent non-linear studies of the tearing mode.

Rutherford (1973), following a suggestion by Kadomtsev and Pogutse (1970), showed that a quasilinear modification of the basic current profile  $\delta J_{0z} \approx V_{y1} B_{x1}/\eta$  in sheet geometry (Figure 2) associated with the growth of the tearing perturbation leads to a third order non-linear force

$\delta J_{0z} B_{x1}/c$  which dominates the linear inertial force  $\rho \partial V_{1y}/\partial t$

in the momentum equation. Rutherford then replaced the inertial force term with the nonlinear force, proceeded with the normal mode analysis and found the similarity solution  $B_{x1} \approx (k B'_{y0}) \eta^2 \Delta'^2 t^2$ ,

where  $B'_{y0} = dB_{y0}/dx$  and  $\Delta'$  is the jump condition in the logarithmic derivative of  $B_{x1}$  at  $\underline{k} \cdot \underline{B} = 0$ ; so that non-linearly the tearing mode grows as  $t^2$  rather than exponentially. This result

can be derived by using some simple arguments but first we will contrast the difference between the Sweet-Parker model and Petschek's model for use later.

The Sweet-Parker model shows that for a steady state configuration, the magnetic field convection produced by the inward moving fluid must be balanced by the resistive diffusion; assuming incompressibility of the fluid, Sweet and Parker found

$$u = (\eta V_A / 4\pi L)^{1/2}.$$

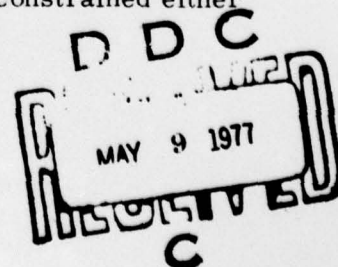
Thus in the Sweet-Parker model resistivity dominates the rate at which the magnetic field can be taken into the neutral regions and hence the rate of magnetic field thermalization.

Petschek showed that the field thermalization problem is more properly treated as a field reconnection problem. His results are as follows:

- . Field reconnection signifies field annihilation also, because  $\underline{B}$  and plasma kinetic energy suffer changes at the shock fronts envisioned by Petschek.
- . Any fluid flow up to  $u \approx .1 V_A$  is permitted.
- . Resistivity is only important in a small region about the x-type point whose size adjusts itself to accommodate any flow (up to  $u \approx .1 V_A$ ) provided by the boundary conditions.
- . Field thermalization is not limited by resistivity but by the fluid flow velocity.

Following Kaw (1976), we align several Petschek boxes end to end to form a periodic magnetic island structure where the reconnection rate and therefore the growth of the magnetic island may be constrained either

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by resistivity or by fluid flow towards the singular layer. Using this periodic island structure we first relate the non-linear results of the tearing mode due to Rutherford (1973) with that of the Sweet-Parker model both of which are resistivity dominated.

Following Kaw (1976) assume a black box of length  $L$  and width  $\delta$  surrounding the x-type neutral point (Fig. (3)). Plasma will flow into the box with velocity  $u$  across the interface having length  $L$ , and then flow along the field lines into each island and cause it to expand. Assuming incompressibility, (which is a good approximation since those components of  $\underline{B}$  that do not vanish make the effective pressure high even though the kinetic pressure is small), the rate of island width growth is given by

$$\frac{dW}{dt} \approx uL/\lambda, \quad (1)$$

where  $\lambda (\approx 1/k)$  is the length of the island,  $W$  the island width and  $u$  is evaluated at  $\delta/2$  such that

$$u \approx E/(\delta B'_{y0}), \quad (2)$$

where we will assume along with FKR that resistivity is only important around  $\underline{k} \cdot \underline{B} = 0$ . Using the geometrical relation

$$L/\delta \approx \lambda/W \quad (3)$$

and Maxwell's equation

$$E/\lambda \approx \partial B_{x1}/\partial t \quad (4)$$

we obtain, using (1) and (2)

$$W \approx [B_{x1} \lambda / B'_{y0}]^{1/2}, \quad (5)$$

which is the standard result for an island width (Spicer, 1976; Kadomtsev and Pogutse, 1970). Ampere's Equation together with Ohm's Law across the resistive singular layer gives

$$J_z = (\nabla \times \underline{B})_z = B_{y0}/a \approx E/\eta \approx \lambda \partial B_{x1} / \partial t, \quad (6)$$

since  $\nabla \times \underline{B} \approx 0$  within the singular layer.

Using the fundamental assumption of Rutherford that the entire island remains within the resistivity dominated layer which implies  $\delta \approx W$ , equation (5), and the fact that  $B'_{y0} \approx B_{y0}/W$  within the resistive layer we find that

$$B_{y1} \approx B'_{y0} \eta^2 t^2 / a^2 \lambda. \quad (7)$$

Since  $\lambda \approx 1/k$  and  $\Delta' \approx 1/a$  we reach, along with Kaw, Rutherford's result.

We can obtain the linear growth rate of the tearing mode by using the fact that in the linear approximation the island width is much less than the resistive layer width. Thus taking  $L \approx \lambda$  and using (1), (2) and (4) we have

$$\frac{dW}{dt} \approx u \approx E / (B'_{y0} \delta) \approx \frac{\partial B_{x1}}{\partial t} \frac{\lambda}{(B'_{y0} \delta)},$$

hence

$$W \approx B_{x1} \lambda / (B'_{y0} \delta)$$

Using

$$\lambda / \eta \frac{\partial B_{x1}}{\partial t} \approx (B_{y0}/a) \approx (B'_{y0} W/a)$$

we find

$$\frac{\dot{B}_{x1}}{B_{x1}} \approx \eta / (a \delta) \quad (8)$$

Balancing the torque due to linear  $\underline{J} \times \underline{B}$  forces against linear inertial forces gives  $\delta \approx (\gamma \rho \eta)^{1/4} (k B'_{y0})^{1/2}$ ,

where  $\gamma = \dot{B}_{x1}/B_{x1}$ , so (8) yields the usual linear growth rate. Thus we find for linear theory  $W \ll \delta$  while for quasi-linear theory  $W \approx \delta$ . The third possibility  $W \gg \delta$  arises with plasma inflow toward the singular layer.

So far we have seen that the Rutherford theory is analogous to the Sweet-Parker theory in that both are dominated by resistivity in the reconnection region. While this is a valid result for a naturally evolving tearing mode the question arises - what occurs when plasma is forced into the region of reconnection with velocity  $u$ , as is the case for the Petschek mechanism? In this case, reconnection and magnetic island growth occur because of reconnection across the shock waves in the ideal region in addition to the small region around the resistive layer. The island width is then determined by the flow velocity  $u$  given by the external boundary conditions so that the island rate of growth is again given by

$$dW/dt \approx uL/\lambda,$$

where  $L$  is the width of the shock interface. Again assuming  $L \approx \lambda$  we obtain

$$W \approx ut$$

so using

$$W \approx (B_{x1} \lambda / B'_{y0})^{1/2}$$

we find

$$B_{x1} = B'_{y0} u^2 t^2 / \lambda,$$

thus island width is determined by the external boundary conditions on the flow velocity and not the resistivity.

In an arch model, as proposed by Spicer (1976), this flow velocity would be in the radial direction so that twisting of the feet of the arch or the propagation of a torsional wave along the arch should result in such a fluid flow, thereby increasing the rate of field thermalization over that discussed by Spicer (1976), and without having to invoke the phenomenon of resonant overlap to be discussed below.

The results presented above support the conclusion of Sturrock (1966), i.e., the non-linear evolution of the tearing mode by the Petschek mechanism, may occur.

Since the island widths can exceed the resistivity-dominated width if fluid dominated and more than one singular layer can occur in a sheared field, the question naturally arises "What would occur if these additional singular layers were to interact, i.e., overlap one another.?"

This question can be answered by recalling that the tearing mode is an instability that requires destroying flux surfaces; that is, the adiabatic invariant of ideal MHD theory is violated, or

$$\frac{d\Phi}{dt} \neq 0, \quad (11)$$

where  $\Phi = \oint \underline{B} \cdot d\underline{S}$ . This can be made clearer for an arch model by converting the field line equations in cylindrical geometry

$$\frac{dr}{B_r} = \frac{rd\phi}{B_\phi} = \frac{dz}{B_z} \quad (12)$$

into canonical form. Imposing helical symmetry through the variable  $\xi = kz - m\phi$ , using  $\underline{\nabla} \cdot \underline{B} = 0$ , and using the

stream function  $\psi$  we obtain

$$B_r = \frac{1}{r} \frac{\partial \psi}{\partial \xi} \quad (13)$$

and

$$B_\xi = -\frac{1}{r} \frac{\partial \psi}{\partial r}, \quad (14)$$

where  $\partial \psi / \partial r = m B_\phi - kr B_z$  and  $\psi$  represents the magnetic flux through a helical ribbon defined by the magnetic axis and a helix of constant phase  $\xi$  and  $r$ . Using  $\rho = r^2/2$  and  $d\phi/dt = (k - d\xi/dz)/m$  the field line equations become

$$\frac{d\rho}{dt} = \frac{\partial \psi}{\partial \xi} \quad (15)$$

and

$$\frac{d\xi}{dt} = -\frac{\partial \psi}{\partial \rho}, \quad (16)$$

where  $\psi$  represents the Hamiltonian,  $\rho$  the conjugate coordinate and  $\xi$  the conjugate momentum, and we have used  $dz/dt = B_z$ . The adiabatic invariants are

$$J = \oint r^2 d\xi / 2 \quad (17)$$

and

$$\frac{d\theta}{dt} = \omega = \frac{\partial \mathcal{H}_0}{\partial J}. \quad (18)$$

$J$  represents the flux tube area and  $\omega$  the frequency of rotation of the flux line around the flux tube.

We note that (15) and (16) can be treated as equations of a non-linear oscillator with singular points  $\partial \psi / \partial r = 0$  and  $\partial \psi / \partial \xi$  i.e.  $\underline{k} \cdot \underline{B} = 0$  and  $B_r = 0$ , respectively. Thus  $\underline{k} \cdot \underline{B} = 0$  represents a spatial resonant surface and in the case of an arch  $\underline{k} \cdot \underline{B} = 0$  occurs when the pitch of  $\underline{B}$  matches the pitch of the perturbation. Indeed  $\omega$  can be written as

$$\omega = \frac{nB_\phi}{r} \left[ q - \frac{m}{n} \right] = n\omega_0. \quad (19)$$

As is well known, a non-linear oscillator's frequency is amplitude dependent. Because of this, a non-linear oscillator's frequency varies when a resonant perturbation acts on it, due to the frequency dependence of the driving force amplitude. Thus the amplitude and frequency of the oscillator will undergo beating over a finite width  $\Delta\omega$  and  $\Delta b$ , where  $\omega$  is the frequency and  $b$  the amplitude of the driving perturbation. In the case of the tearing mode  $\Delta\omega$  represents the width of a magnetic island and  $\Delta b$  the spread in the current perturbation magnitude caused by the reconnection process. If we denote  $\epsilon$  as the perturbation amplitude due to the tearing mode it can be shown that the width of the resonance at  $\underline{k} \cdot \underline{B} = 0$ , i.e., the island width, is given by

$$W = (\epsilon d\mu/dr)^{1/2},$$

which is identical to our earlier result, where  $\mu = B_\phi / r B_z$  and

$d\mu/dr$  is the shear.

We can now answer the question - what happens when one or more resonances interact or overlap one another? A definition of resonant overlap is

$$S = W/d,$$

where  $W$  is the width of one of the resonances and  $d$  the distance between them. When  $S < 1$  no interaction occurs (Fig. (4)) and when  $S \gtrsim 1$  interaction occurs (Fig. (5)). When  $S \gtrsim 1$  there is a substantial increase in reconnection because islands are being pushed against one another. In addition, this interaction results in some other interesting phenomena. Suppose the resonances are well separated so that they do not interact. In this case we can superimpose the results of a single resonance. However, if the resonances are close together so that  $S \gtrsim 1$  and the phases of the driving perturbations are random, field lines will undergo Brownian motion in their position, so that a turbulent spectrum of MHD waves may be excited.

Further, because

$$W \approx (\epsilon / d\mu/dr)^{1/2}$$

and the distance between resonance surfaces  $d$  is inversely proportional to the shear it is easy to see that the strength of the perturbation required for overlap becomes weaker with stronger shear (Finn, 1975). We should therefore expect strong overlap in arches with strong shear.

Recent non-linear numerical simulation studies (Schnack and Killeen, 1976) have confirmed that the interaction of two singular layers leads to a substantial increase in the degree of reconnection and the rate of reconnection. The growth rates were found to approach MHD rates when two singular layers overlapped.

If the width of the islands is determined by the flow velocity rather than the resistivity the likelihood of overlap occurring is also greater since flow dominated reconnection results in larger island widths.

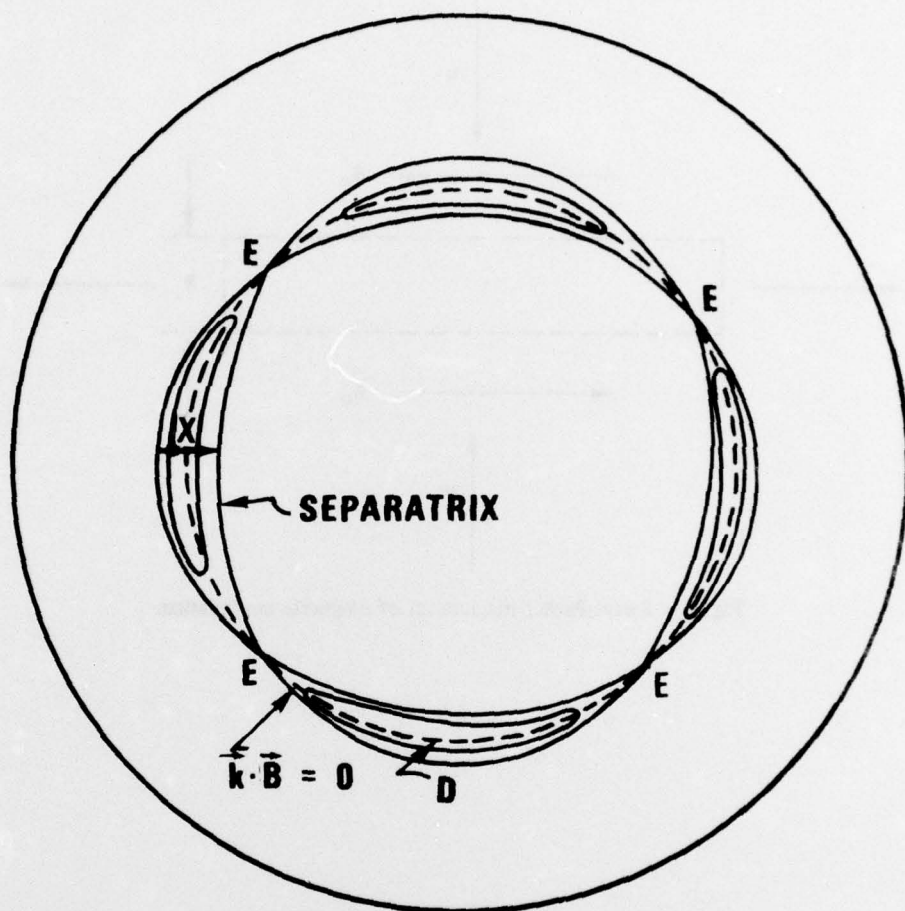
One can draw a number of qualitative predictions for flares if overlap were to occur, they are:

- . The volume of primary flare energy release can be reduced because of the increase in effective reconnecting volume.
- . Regions of strong magnetic shear will lead to flares with faster rise times due to the increase in rate of magnetic field dissipation.

#### ACKNOWLEDGMENTS

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**m = 4 mode**

Fig. 1 — Four magnetic islands

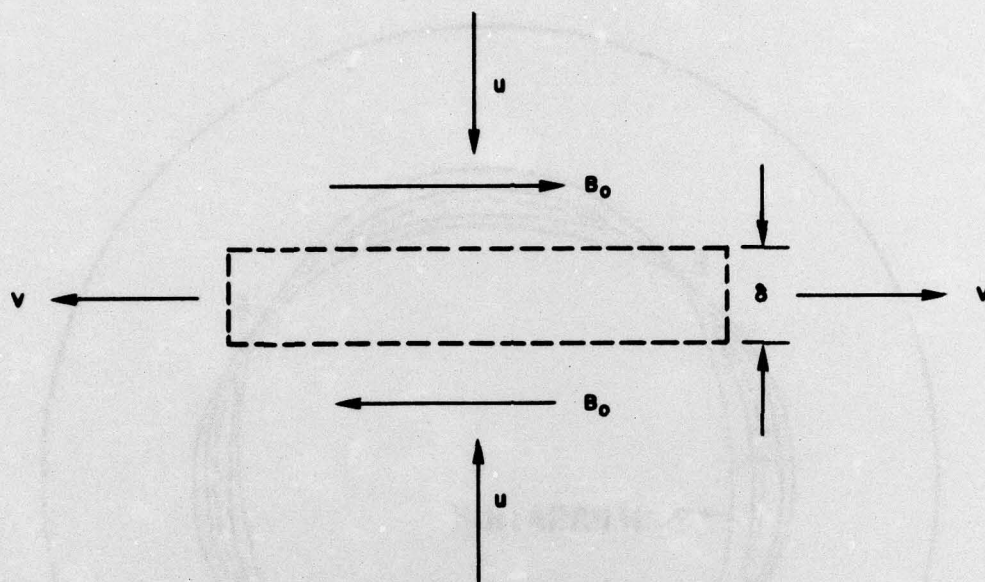


Fig. 2 — Sweet-Parker mechanism of magnetic annihilation

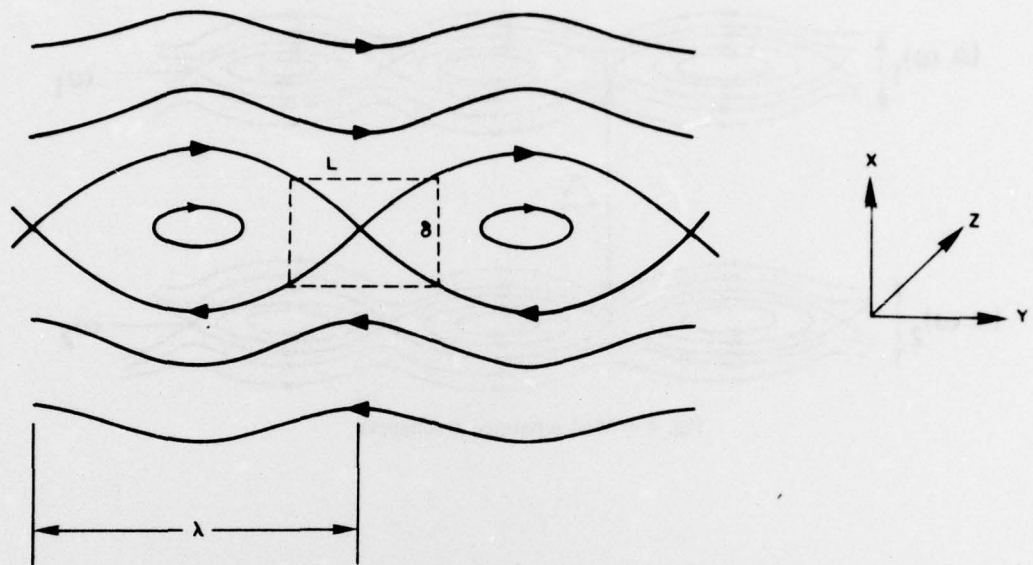


Fig. 3 — Magnetic island growth with a diffusion region near X point

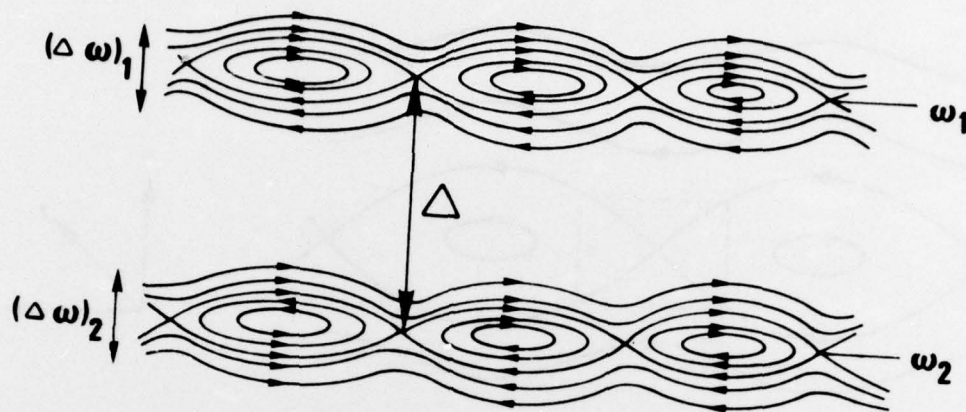


Fig. 4 — Well separated resonances

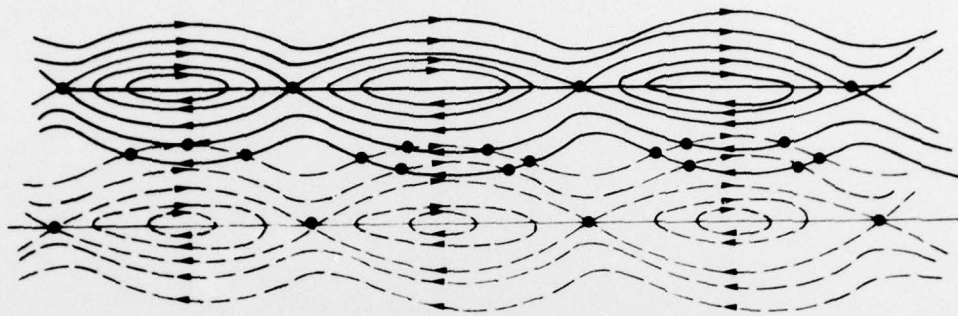


Fig. 5 — Weak nonlinear overlap of resonances  
(note increase in number of neutral points)